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BRIEF COMMUNICATION

A SIMPLE TWO-PHASE FRICTIONAL PRESSURE DROP CALCULATION METHOD

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1. INTRODUCTION

Despite the importance of pressure drop in two-phase flow processes, and the consequent extensive research into the topic, there is still no satisfactory method for calculating two-phase pressure drop. The best current methods are cumbersome in structure, heavily dependent on empirically determined coefficients, and have considerable uncertainty. Simpler forms or firmer theoretical bases for predictive methods can only be achieved with a narrowing of the ranges of applicability.

In this note, a theoretically based, flow pattern dependent calculation method is adapted to yield a simple predictive method in which flow pattern influences are partially allowed for in an implicit manner and therefore need not be explicitly taken into account when using the method.

2. METHOD

It has been shown that pressure drop equations derived from mixing length theory are applicable to two-phase conditions provided flow pattern effects are taken into account (Beattie 1977). However, the correct allowance for flow pattern effects is not understood. If it is assumed that coefficients appearing in mixing-length theory are the same as those in the single-phase application of the theory, loss of generality occurs. However the familiar Colebrook–White equation relating Fanning friction factor f, Reynolds number Re, and surface roughness/diameter ratio ϵ/D, is found:

$$\frac{1}{\sqrt{f}} = 3.48 - 4\log_{10}[2\epsilon/D + 9.35/(\mathrm{Re}\sqrt{f})]\,. \qquad [1]$$

Despite the loss of generality, [1] remains valid for many two-phase flow situations provided flow pattern effects are considered in the definition of the friction factor and Reynolds number (Beattie 1973). Especially relevant to this note are definitions applicable to bubble and annular flows. For these flows, Re and f are the same as in the homogeneous model:

$$f = \frac{\bar{\rho} D\left[\frac{dp}{dz}\right]_F}{2G^2} \qquad [2]$$

$$\mathrm{Re} = \frac{DG}{\mu} \qquad [3]$$

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where $[dp/dz]_F$ is the frictional pressure gradient, G is the mass flux, μ is the two-phase viscosity, and $\bar{\rho}$ is the homogeneous density given by

$$\frac{1}{\bar{\rho}}=\frac{x}{\rho_g}+\frac{1-x}{\rho_l}$$

where x is the gas mass flow fraction (quality), ρ_g is the gas density and ρ_l is the liquid density.

Flow pattern effects are allowed for in the viscosity definition. For many bubble and annular flows, the appropriate definitions are

$$\mu = \mu_l(1+2.5\beta) \quad \text{(bubble flow)} \tag{4}$$

and

$$\mu = \mu_l(1-\beta)+\mu_g\beta \quad \text{(annular flow)} \tag{5}$$

where β is the homogeneous void fraction

$$\beta=\frac{\rho_l x}{\rho_l x+\rho_g(1-x)},$$

μ_l is the liquid viscosity and μ_g is the gas viscosity. [4] and [5] approach μ_l as the void fraction tends to zero.

The bubble flow definition[4], is based on the theoretical work of Einstein (1906). Direct measurements of bubble viscosity (Huige & Ohki 1972) have confirmed the validity of a variation of [4] up to a voidage of about 20%. The equation found valid by Huige & Ohki (1972) reflects the surface chemistry of liquids studied by them, and their results indicate that [4] is applicable for fluids usually encountered in gas–liquid systems. The applicability of [4] to friction factor calculations at very much higher voidage (Beattie 1973 and 1977) appears to be due to an overestimate of the viscous sublayer void fraction, taken as β, being compensated for by the coefficient 2.5, which data suggest is too low at high void fractions.

The annular flow definition, [5], is based on the concept that, due to interfacial waves, the viscosity at the outer region of the viscous sublayer intermittently changes between μ_l and μ_g, with the intermittency fraction (i.e. local voidage) being assumed to be β.

We propose that [4] and [5] be replaced by a hybrid definition

$$\mu = \mu_l(1-\beta)(1+2.5\beta)+\mu_g\beta. \tag{6}$$

The structure of [6] is consistent with the form that might be expected for some gravity dominated flows such as horizontal stratified bubbly flow or vertical bubbly/slug flow. For such flows, it might be expected that the viscosity for annular flows, [5], applies provided μ_l is replaced by [4] to allow for entrained bubbles in the liquid region of the flow.

The assumption that mixing-length theory results apply to horizontal flows is based on the observation (Beattie 1972) that opposite sides of asymetric flow profiles often coincide when normalised with respect to the distance between the wall and the position of zero shear stress.

The proposed method thus involves[1–3, 6]. A special feature of the model is the use of [1] for all values of Re, even for those values where single-phase behaviour would suggest its replacement by a laminar flow relation. This is because turbulent-like characteristics extend to very low Reynolds numbers in two-phase flows. This is perhaps not surprising because even in the absence of turbulence, Reynolds stresses can be expected in two-phase flow due to the interaction of gas–liquid interfaces with the flow field.

3. COMPARISON WITH DATA

Since the proposed method aims to allow approximately for flow pattern effect without specific knowledge of flow pattern boundaries, a proper test of the method should involve a comparison with data covering various flow patterns. Clearly, the method would not, for example, compare as favourably with bubble flow data as well as the bubble flow model which involves [4] instead of [6]. In order to achieve an adequate test of the method, use was made of a proprietary adiabatic, round tube pressure drop data bank covering both horizontal flow and vertical upflow (HTFS 1981). This data bank covers a wide range of fluids and flow conditions.

The results of the comparison are summarised in table 1. Calculation methods commonly used in two-phase flow are also included in table 1. The correction factor of table 1 is the average value of

$$\frac{\text{actual total pressure drop}}{\text{calculated total pressure drop}}$$

and the range factor is the factor by which the corrected calculated value must be multiplied and divided to give the range for 99% probability and 95% confidence. Thus, the accuracy of a particular predictive method is best improved by multiplying the prediction by the correction factor. Multiplying and dividing a prediction thus corrected by the range factor produces a range which has a 95% probability of including 99% of relevant data. The range factor is thus a measure of the spread of errors.

The method proposed here predicts only the frictional component of the pressure drop, whereas the test of the method involves also the estimation of the accelerational pressure drop, and, in the case of upflow, the gravitational pressure drop. The accelerational pressure drop was calculated using the homogeneous model. The number of data points in the bank with significant contributions from the accelerational head are small enough for error statistics of table 1 not to be influenced by errors in the estimate of the accelerational head.

On the other hand, sufficient of the vertical flow data of the data bank had significant gravitational pressure drop contributions for the error statistics of table 1 to be influenced by

Table 1. Comparison of pressure loss correlations with HTFS (1981) data bank

Frictional Pressure Drop Correlation	Non Steam/Water				Steam/Water			
	Horizontal		Vertical		Horizontal		Vertical	
	Correction Factor	Range Factor	Correction Factor	Range Factor	Correction Factor	Range Factor	Correction Factor	Range Factor
HTFS (1981)	1.00	2.6	1.13	2.0	0.82	2.4	0.97	1.5
Baroczy (1966)	0.92(1)	2.4	1.17(1)	2.2	0.77(1)	2.8	0.97(1)	1.5
Friedel (1979)	0.77(1)	4.0	1.01(1)	2.0	0.85	2.4	0.97	1.6
Lockhart-Martinelli (1949)	0.99	2.8	0.96	1.9	0.43	3.9	0.59	3.3
Martinelli-Nelson (1948)					0.58	2.6	0.76	2.2
Thom (1964)					0.87(1)	2.4	0.93(1)	1.7
Chisholm (1978)	0.74	4.8	0.94	2.1	0.68	2.9	0.92	1.6
Isbin et al (1954)	1.07	3.8	2.00	3.8	0.94	2.6	1.16	2.2
Owens (1961)	0.53	9.1	1.39	4.6	0.85	2.6	1.10	2.2
Cicchitti et al (1960)	0.61	6.2	1.53	4.1	0.89	2.6	1.12	2.2
Dukler et al (case 2) (1964)	1.18	3.1	2.08	3.9	0.97	2.6	1.20	2.2
Equations 1,2,3,6	1.05	3.1	1.16	2.1	0.94	2.5	1.01	1.7
Number of data points	7168		2011		1236		3095	

Note: (1) indicates that, due to limitation on the correlation applicability range, comparison has been made with a reduced number of data.

the gravitational pressure drop calculation method. Since the Isbin *et al.* (1954), Owens (1961), Cicchitti *et al.* (1960) and Dukler *et al.* (1964) methods can be regarded as variations of the homogeneous model, the gravitational head for these models was calculated using the homogeneous void fraction model. The gravitational head for the other pressure loss models was calculated using an unpublished void fraction correlation (HTFS 1981), an independent examination having shown this to be the most reliable for the range of conditions covered by the data bank.

Although the error statistics for the vertical flow cases of table 1 depend on the gravitational pressure drop calculation method, the use of an apparently reliable gravitational method or, in some cases, a gravitational method consistent with the frictional method, means that the error statistics for vertical uplow can be primarily associated with errors in the frictional methods.

4. DISCUSSION

Table 1 indicates that, despite its simplicity, the proposed frictional pressure drop calculation method generally gives more accurate results than alternatives tested apart from the HTFS correlation. At first sight this is unexpected: the proposed method gives a two-phase multiplier approximately independent of mass flux, whereas this multiplier is known to be strongly influenced by mass flux. Indeed, the HTFS, Baroczy (1966), Friedel (1979) and Chisholm (1978) correlations specifically allow for this effect. However from an examination of the data it seems probable that a large contribution to the range factors occurs because of the inability to allow for effects such as entrance conditions which are known to effect two-phase pressure drop. The comparatively good performance of the present method suggests that errors due to the neglect of mass flux effects are not as significant as those due to the neglect of other effects.

The present method has been tested only against adiabatic, round tube, data. It has not been compared with diabatic or complex geometry data, which are known to have different pressure drop characteristics (Beattie 1977). However, the characteristics for these systems are similar to those for round tube adiabatic flows, particularly at low void fraction, and it is expected that application of the present model to diabatic and complex geometry systems should give reasonable results though with a larger error than for adiabatic round tube flows. The larger error will arise, for example, from additional flow disturbance effects, such as from spacers in rod bundles. The present model underpredicts condensation pressure drop by a considerable margin. A preliminary comparison with condensation data indicates that [1–4] give reasonable estimates of pressure drop during condensation.

5. SUMMARY AND CONCLUSIONS

A mixing-length theory model of two-phase pressure drop has been adapted to give a simple calculation method, of the same form as that used for single phase flow, in which flow pattern effects are partially accounted for implicitly. Comparison of the method with an extensive adiabatic round tube data bank shows it to be as good as most alternative, more complex, methods. Errors in the present method are due primarily to the neglect of effects such as entrance effects rather than the neglect of mass flux effects, so methods which more correctly allow for mass flow effects do not necessarily result in more reliable predictions. Except for condensing flows, the method should also yield reasonable pressure loss estimates for diabatic and/or complex geometry flows.

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